

Physics 831, Statistical Physics
Fall 2001

Homework Set#5 (Assigned: September 28, 2001)

- 5.1) For a system described by a canonical ensemble (constant T, N, V) show that

$$\langle (H - \langle H \rangle)^2 \rangle = k_B^2 [T^2 \left(\frac{\partial C_V}{\partial T} \right)_V + 2T^3 C_V],$$

where $\langle A \rangle$ is the canonical ensemble average of a physical quantity A. (7 points)
 For an ideal mono-atomic gas, show that (3 points)

$$\frac{\langle (H - \langle H \rangle)^2 \rangle}{\langle H \rangle^2} = \frac{8}{9N^2}$$

- 5.2) Consider a system of N atoms, each of which has an intrinsic magnetic moment of magnitude μ . The Hamiltonian in the presence of an external magnetic field \mathbf{B} is

$$H(q, p) - \mu B \sum_{i=1}^N \cos \theta_i,$$

where $H(q, p)$ is the Hamiltonian of the system in the absence of the external magnetic field q and p being the usual generalized coordinates and momenta, and θ_i is the angle between \mathbf{B} and the magnetic moment of the i^{th} atom.

- (a) Show that the average induced magnetic moment M in the direction of the external field is (8 points)

$$M = N\mu \left(\coth \alpha - \frac{1}{\alpha} \right), \quad \left(\alpha = \frac{\mu B}{k_B T} \right)$$

The parameter α is a ratio of magnetic energy (μB) to thermal energy ($k_B T$). Note that M does not depend on the field independent part of the total Hamiltonian.

- (b) Show that the isothermal magnetic susceptibility $\chi_T = (\partial M / \partial B)_T$ per atom is (3 points)

$$\chi_T = \frac{\mu^2}{k_B T} \left(\frac{1}{\alpha^2} - \text{csc}^2 \alpha \right)$$

- (c) At high temperature i.e. when the thermal energy is much larger than the magnetic energy, the above susceptibility satisfies Curie's law, namely $\chi_T \propto T^{-1}$. Find the proportionality constant, which is called Curie's constant. (4 points)